F.Y.B.Sc. (IT)(Sem-I) April-2023

Q.P. Code:00000718

Discrete Mathematics

[Time:2.30 Hrs] **[Marks:75]**

Please check whether you have got the right question paper.

N.B:
1. All question are compulsory.
2. Figures to the right indicate full marks.
3. Students answering in the regional language should refer in case of doubt to the main text of the paper in English.

Q.1 Attempt **any three** of the following: 15

a.) Let A = {c, d, f, g}, B = {f, j}, and C = {d, g}.

Answer each of the following questions. Give reasons for your answers.

a. Is $B \subseteq A$?
b. Is $C \subseteq A$?
c. Is $C \subseteq C$?
d. Is C a proper subset of A?

b.) Let S = {4, 6, 8} and T = {1, 3, 5}. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

a. $S \times T$
b. $T \times S$
c. $S \times S$
d. $T \times T$

c.) Write negations for each of the following statements.

(Assume that all variables represent fixed quantities or entities, as appropriate.)

i. If P is a square, then P is a rectangle.

ii. If today is New Year's Eve, then tomorrow is January.

iii.If n is prime, then n is odd or n is 2.

iv. If x is nonnegative, then x is positive or x is 0.

v. If n is divisible by 6, then n is divisible by 2 and n is divisible by 3.

d.) Let A = {a, b, c}, B = {b, c, d}, and C = {b, c, e}.

1. Find $A \cup (B \cap C)$, $(A \cup B) \cap C$, and $(A \cup B) \cap (A \cup C)$. Which of these sets are equal?

2. Find $(A - B) - C$ and $A - (B - C)$. Are these sets equal?

e.) Explain Russell's Paradox.

f.) Test the validity of the following argument.

If I study, then I will not fail mathematics.

If I do not play basketball, then I will study.

But I failed mathematics.

Therefore I must have played basketball.

Q.2 Attempt **<u>any three</u>** of the following: 15

a.) Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$.

Determine which of the following statements are true and which are false. Provide counter examples for those statements that are false.

a. $\forall x \in D$, if x is odd then $x > 0$.
b. $\forall x \in D$, if x is less than 0 then x is even.
c. $\forall x \in D$, if x is even then $x \le 0$.
d. $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.
e. $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

b.) Rewrite the statement "No good cars are cheap" in the form "$\forall x$, if $P(x)$ then $\sim Q(x)$." Indicate whether each of the following arguments is valid or invalid, and justify your answers.

a. No good car is cheap.
   A Rimbaud is a good car.

   ∴ A Rimbaud is not cheap.

b. No good car is cheap.
   A Simbaru is not cheap.

   ∴ A Simbaru is a good car.

c. No good car is cheap.
   A VX Roadster is cheap.

∴ A VX Roadster is not good.

d. No good car is cheap.
An Omnex is not a good car.

∴ An Omnex is cheap.

c.) Prove that √5 is irrational.

d.) Prove that for all integers n, if n > 2 then there is a prime number p such that n < p < n!.

e.) Prove that for all integers m and n, m + n and m – n are either both odd or both even.

f.) Prove that for all integers n, $n^2 - n + 3$ is odd.

Q.3 Attempt **<u>any three</u>** of the following: 15

a.) Prove that n! + 2 is divisible by 2, for all integers n ≥ 2.

b.) Prove that 7n − 1 is divisible by 6, for each integer n ≥ 0.

c.) Find the first four terms of each of the recursively defined sequence

Sk = Sk-1 + 2Sk−2, for all integers k ≥ 2 S0 = 1, S1 = 1

d.) Prove or give counter examples for the following

i) If f : X → Y and g: Y → Z are functions and g ◦ f is one-to-one, must g be one-to-one?

ii) If f : X → Y and g: Y → Z are functions and g ◦ f is onto, must f be onto?

e.) List and explain different types of functions. (any 5)

f.) Define g: Z → Z by the rule g(n) = 4n − 5, for all integers n.

(i) Is g one-to-one? Prove or give a counter example.

(ii) Is g onto? Prove or give a counter example.

Q.4 Attempt **<u>any three</u>** of the following: 15

a.) Define the following:

a) Trail b) Connected Graph c) Spanning Tree d) Hamiltonian Graph e) Hamiltonian Cycle

b.) Explain Dijkstra’s Algorithm with the help of example.

c.) i) Let R = {(0, 1), (0, 2), (1, 1), (1, 3), (2, 2), (3, 0)}. Find R$, the transitive closure of R.

ii) Let S = {(0, 0), (0, 3), (1, 0), (1, 2), (2, 0), (3, 2)}. Find St , the transitive closure of S.

iii) Let T = {(0, 2), (1, 0), (2, 3), (3, 1)}. Find T t , the transitive closure of T .

d.) The relation R is an equivalence relation on the set A. Find the distinct equivalence classes of R.

i) A = {0, 1, 2, 3, 4}

R = {(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3),(4, 0), (4, 4)}

ii) A = {a, b, c, d}

R = {(a, a), (b, b), (b, d), (c, c), (d, b), (d, d)}

iii) A = {1, 2, 3, 4, . . . , 20}. R is defined on A as follows:

For all x, y ∈ A, x R y ⇔ 4 | (x − y).

e.) Let A = {1,2,3,4} R be a relation on set A defined by

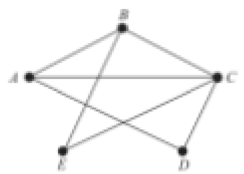
R = {(1,1,), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (3,4), (4,4)}

Construct the matrix and digraph of R.

f.) Consider the following graph G

(a) Describe G formally, that is, find the set V (G) of vertices of G and the set E(G) of edges of G.

(b) Find the degree of each vertex and verify Handshaking Lemma Theorem for this graph.



Q.5 Attempt **any three** of the following: 15

a.) i. How many five-digit integers (integers from 10,000 through 99,999) are divisible by

5?

ii. What is the probability that a five-digit integer chosen at random is divisible by 5?

b.) The instructor of a discrete mathematics class gave two tests. Twenty-five percent of the students received an A on the first test and 15% of the students received A's on both tests. What percent of the students who received A's on the first test also received A's on the second test?

c.) An urn contains four balls numbered 2, 2, 5, and 6. If a person selects a set of two balls at random, what is the expected value of the sum of the numbers on the balls?

d.) A pool of 10 semi finalists for a job consists of 7 men and 3 women. Because all are considered equally qualified, the names of two of the semi finalists are drawn, one after the other, at random, to become finalists for the job.

a. What is the probability that both finalists are women?

b. What is the probability that both finalists are men?

c. What is the probability that one finalist is a woman and the other is a man?

e.) Two cards are drawn at random from an ordinary deck of 52 cards. Find the probability p that:

(a) both are spades;

(b) one is a spade and one is a heart.

f.) Define the following:

a. Event
b. Experiment
c. Probability
d. Addition Probability
e. Multiplication Probability

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